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ON THE LIMIT OF PLANETARY STABILITY.

BY PROFESSOR DANIEL KIRKWOOD.

LAPLACE, in his *Syste\`me du Monde,* pointed out the limit at which, according to his estimate, the moon's attraction could have retained an elastic atmosphere.* The question of a satellite's stability was also considered by the late Professor Vaughan, of Cincinnati.† I have seen no attempt, however, to obtain for the different members of our system any definite numerical results. In the present paper it is proposed to find the approximate limits of stability in the cases of the eight major planets and certain of the satellites, on the hypothesis that their primitive condition was either liquid or gaseous.

Let $M =$ the mass of the larger or central body,
$m =$ that of the dependent planet or satellite,
$x =$ the distance from the centre of the former to the limit of stability of the latter,
$a =$ the distance between their centres; then, since the disturbing or separating force of the larger upon the smaller mass is the difference between the attraction of the former on the nearest point of the surface of the latter and that on its centre of gravity, we have

$$\frac{M}{x^2} - \frac{M}{a^2} = \frac{m}{(a-x)^2}, \qquad (1)‡$$

or, putting $a = 1$ and reducing,

$$x^4 - 2x^3 + \frac{m}{M}x^2 + 2x = 1. \qquad (2)$$

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*Syst. du Monde, B. IV., Ch. X.

†Pop. Sci. Monthly for Sept, 1878. See also the Proc. of the A. A. A. S. for 1856.

‡We neglect the centrifugal force due to the planet's rotation, as the modification would be slight and we propose to obtain merely approximate results.

If we adopt the masses and distances given in Newcomb's Popular Astronomy and solve equation (2) for each of the eight principal planets we shall obtain the distance from the centre of each to its limit of stability, as given in the second column of the following table. If, moreover, the planets, with their present masses, be reduced to the sun's mean density their radii as stated in the third column are found by the formula

$$r_n = 430{,}000\left(\frac{m_n}{M}\right)^{\frac{1}{3}},$$

and the respective ratios of the limits of stability to these radii are seen in column fourth.

TABLE.

Planet.	R_n	r_n	$\frac{R_n}{r_n}$
Mercury	165,165 ms	2,514.6 ms	65.7
Venus	701,746	5,719.2	122.7
Earth	1,059,386	6,242.7	169.7
Mars	764,900	2,951.1	259.2
Jupiter	37,354,287	42,335	882.35
Saturn	45,859,381	28,317	1619.48
Uranus	49,512,900	15,209	3255.51
Neptune	81,663,510	16,009	5101.10

On the assumption that in each case the mean density of the separated mass was equal to that of the central body, the sun's present radius multiplied by the respective numbers in column fourth will give the radii of the solar nebula when the planets extended to their respective limits of stability. These radii are less than the mean distances of the planets in the ratio of 1 to 1.265. This fact may have some significance in regard to the former oblateness of the solar nebula or the law of its density.

The Earth and the Moon.—For the moon, which in perigee approaches within 221500 miles of the earth, the limit of stability is about 38000 miles. Were the moon's density reduced to that of the earth its radius would be 916 miles, the ratio of which to the limit of stability is 1 : 41.6. The moon's least distance diminished by 38000 miles is 183500 miles. If our satellite originally extended to the limit, and if the moon and the earth had the same form and density, the radius of the latter was 165000 miles.

The Martian System.—The diameter of Phobos, according to Prof. Pickering, is 5.57 miles. If its density, therefore, be equal to that of Mars the limit of stability is about two miles exterior to the surface; or, if the density be to that of the primary in the same ratio as the density of the moon to that of the earth, the limit is less than a mile from the surface of the satellite;

and finally if the density were no greater than that of water the satellite, if fluid, would be unstable, the limit being actually within the surface. Since, therefore, the satellite could never have existed at its present distance in a nebular state, it must follow, if any form of the nebular hypothesis is to be accepted, that its original distance was much greater than the present. Can we find a probable cause for this ancient disturbance?

If we suppose the former period of Mars to have been very nearly one-sixth that of Jupiter the close commensurability would render the orbit of Mars more and more eccentric. The planet in perihelion would thus pass through the sun's atmosphere, or rather through the outermost equatorial zone of the solar nebula. This resisting medium would not only accelerate the motion of Mars but also in a much greater degree that of his extremely small satellite. The solar mass contracting more rapidly than the orbit of Mars would finally leave the latter moving in an excentric path without sensible resistance.

Other Secondary Systems.—For the first satellite of Jupiter the limit is 5250 miles, or 4½ times the radius of the satellite. For Mimas, the innermost satellite of Saturn, it is less than twice the radius. The rings of Saturn, in all probability, could not exist as three satellites, the limits of stability being interior to the surface.*

The effect of perturbation in the dismemberment of comets is known to all astronomers. The nucleus of the great comet of 1880, which approached within less than 100,000 miles of the sun's surface, must have had a density greater than that of granite, as well as a strong cohesive force betw'n its parts, in order to withstand the tendency to disintegration during its perihelion passage. Had the nucleus been either liquid or gaseous, or even a cluster of solid meteorites, the difference between the sun's attraction on the central and the superficial parts would have pulled the comet asunder, spreading out the fragments into somewhat different orbits, like the meteoric streams of August and November.

LAW OF FACILITY OF ERRORS IN TWO DIMENSIONS.

BY E. L. DE FOREST, WATERTOWN, CONN.

In a recent paper (Analyst, Nov., 1880) I gave some account of what is known as Lagrange's theorem in probability, and showed how it can be

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*It has been recently shown that Bessel's mass of the ring is much greater than the true value.